4. Soustavy lineárních rovnic s jednou neznámou

Příklady

① Řešte v R

- vyřešíme každou rovnici zvlášť
- MNOŽINA VŠECH ŘEŠENÍ $\mathcal{K}$ = PRŮNIK množin všech řešení $\mathcal{K}_1, \mathcal{K}_2, \ldots$ jednotlivých rovnic

$\mathcal{K} = \mathcal{K}_1 \cap \mathcal{K}_2$

a) $\begin{array}{l} 2x-7 \leq 0 \\ 3x+1 > 0 \end{array}$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

a současně

$2x-7 \leq 0 \ \wedge \ 3x+1>0$

$2x \leq 7$ $\qquad 3x > -1$

$x \leq \frac{7}{2}$ $\qquad x > -\frac{1}{3}$

$\mathcal{K}_1 = (-\infty, \frac{7}{2}\rangle$ $\qquad \mathcal{K}_2 = (-\frac{1}{3}, +\infty)$

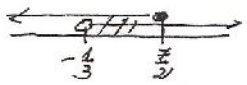
průnik

$\mathcal{K} = \mathcal{K}_1 \cap \mathcal{K}_2$

$\mathcal{K} = (-\frac{1}{3}, \frac{7}{2}\rangle$

ZKRÁCENĚ

$x \leq \frac{7}{2} \ \wedge \ x > -\frac{1}{3}$



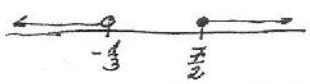
$\mathcal{K} = (-\frac{1}{3}, \frac{7}{2}\rangle$

b) $\begin{array}{l} 2x-7 \geq 0 \\ 3x+1 < 0 \end{array}$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

$2x-7 \geq 0 \ \wedge \ 3x+1<0$

$2x \geq 7$ $\qquad 3x < -1$

$x \geq \frac{7}{2}$ $\qquad x < -\frac{1}{3}$



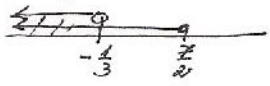
$\mathcal{K} = \emptyset$ (průnik)

c) $\begin{array}{l} 2x-7 \leq 0 \\ 3x+1 < 0 \end{array}$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

$2x-7 \leq 0 \ \wedge \ 3x+1<0$

$2x \leq 7$ $\qquad 3x < -1$

$x \leq \frac{7}{2}$ $\qquad x < -\frac{1}{3}$



$\mathcal{K} = (-\infty, -\frac{1}{3})$

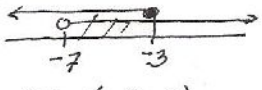
② Řešte v R

a) $-2 < y+5 \leq 2$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

$-2 < y+5$ $\qquad y+5 \leq 2$

$-7 < y$ $\qquad y \leq -3$

$y > -7$



v R: $\mathcal{K} = (-7, -3\rangle$

[kdyby v Z: $\mathcal{K} = (-7, -3\rangle \cap Z$ vybrat z vypočítaného intervalu jen celá čísla

$\mathcal{K} = \{-6, -5, -4, -3\}$

nebo i

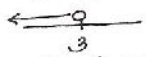
$-2 < y+5 \leq 2 \quad /-5$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

$-7 < y \leq -3$

b) $2x < x+3 < x+10$ $\quad \mathcal{O}=R$, $\mathcal{D}=R$

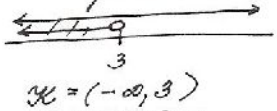
$2x < x+3 \ \wedge \ x+3 < x+10$

$x < 3$ $\qquad 0x < 7$ platí $\mathcal{K}_2 = \mathcal{D} = R$



$\mathcal{K}_1 = (-\infty, 3)$

průnik



$\mathcal{K} = (-\infty, 3)$

c) $-3 \leq x-2 < 2x+1 < \frac{1}{2}x$ $\quad \mathcal{O}=\mathbb{R}$ $\quad \mathcal{D}=\mathbb{R}$

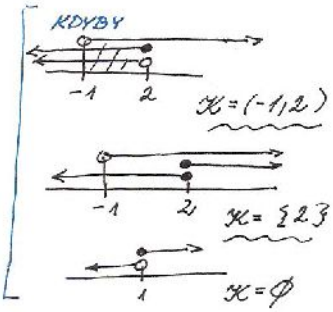
$-3 \leq x-2 \;\wedge\; x-2 < 2x+1 \;\wedge\; 2x+1 < \frac{1}{2}x \quad /\cdot 2$

$-1 \leq x$ $\qquad -x < 3 \quad /\cdot(-1)$ $\qquad 4x+2 < x$

$x \geq -1$ $\qquad x > -3$ $\qquad 3x < -2$

$(\mathcal{K}_1)$ $\qquad (\mathcal{K}_2)$ $\qquad x < -\frac{2}{3}$ $\quad (\mathcal{K}_3)$

KDYBY



$\mathcal{K} = \mathcal{K}_1 \cap \mathcal{K}_2 \cap \mathcal{K}_3$

[pozor na pořadí osobů, např. záporných]

$\mathcal{K} = \langle -1, -\frac{2}{3})$

③ Určete definiční obor

a) $\sqrt{2x - \frac{2x-11}{4} - \frac{19-2x}{2}} + \sqrt{\frac{x}{3} + \frac{x-1}{3} - \frac{2x+15}{9}}$

VÝRAZY POD DRUHOU ODMOCNINOU MUSÍ BÝT ≥ 0 (protože $\sqrt{a}$ je definován pro $a \geq 0$)

$\mathcal{D}$: $2x - \frac{2x-11}{4} - \frac{19-2x}{2} \geq 0 \quad /\cdot 4 \;\wedge\; \frac{x}{3} + \frac{x-1}{5} - \frac{2x+15}{9} \geq 0 \quad /\cdot 45$

$8x - (2x-11) - 2(19-2x) \geq 0$

$8x - 2x + 11 - 38 + 4x \geq 0$

$10x - 27 \geq 0$

$10x \geq 27$

$x \geq \frac{27}{10}$

$x \geq 2{,}7$

$15x + 9(x-1) - 5(2x+15) \geq 0$

$15x + 9x - 9 - 10x - 75 \geq 0$

$14x - 84 \geq 0$

$14x \geq 84$

$x \geq \frac{84}{14}$

$x \geq 6$

POZOR - nedělejme průnik def. obor $\mathcal{D}$

$\mathcal{D} = \langle 6, +\infty)$

b) $\sqrt{\frac{3-11x}{4} - 6x + 8} + \frac{1}{\sqrt{(x+2)^2 - x(x+2)}}$ $\quad \mathcal{O}=\mathbb{R}$

$\mathcal{D}$: $\frac{3-11x}{4} - 6x + 8 \geq 0 \quad /\cdot 4 \;\wedge\; (x+2)^2 - x(x+2) \geq 0 \;\wedge\; (x+2)^2 - x(x+2) \neq 0$

$3 - 11x - 24x + 32 \geq 0$

$-35x + 35 \geq 0$

$-35x \geq -35 \quad /\cdot(-1)$

$35x \leq 35 \quad /:35$

$x \leq 1$

$(x+2)^2 - x(x+2) > 0$

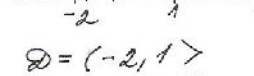
$x^2 + 4x + 4 - x^2 - 2x > 0$

$2x + 4 > 0$

$2x > -4$

$x > -2$

průnik



$\mathcal{D} = (-2, 1\rangle$

c) $\sqrt{\frac{11-4x}{5} - 4x - 7} + \frac{1}{\sqrt{(x+3)^2 - x(x-3)}}$

$\mathcal{D}$:

$\frac{11-4x}{5} - 4x - 7 \geq 0 \quad | \cdot 5 \quad \wedge \quad (x+3)^2 - x(x-3) \geq 0 \quad \wedge \quad (x+3)^2 - x(x-3) \neq 0$

$11 - 4x - 20x - 35 \geq 0$

$-24x - 24 \geq 0$

$-24x \geq 24 \quad | \cdot (-24)$

$x \leq 1$

$(x+3)^2 - x(x-3) > 0$

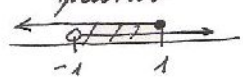
$x^2 + 6x + 9 - x^2 + 3x > 0$

$9x + 9 > 0$

$9x > -9$

$x > -1$

průnik



$\mathcal{D} = (-1, 1\rangle$

④ řeším soustavu rovnic a nerovnic

$x - 2 = \frac{2-3x}{8} + \frac{8-x}{12} \quad | \cdot 24 \quad \wedge \quad x^2 + 2x - 1 > (x-2)^2 - 3$

$24x - 48 = 3(2-3x) + 2(8-x)$

$24x - 48 = 6 - 9x + 16 - 2x$

$24x - 48 = 22 - 11x \quad | + 11x + 48$

$35x = 70 \quad | : 35$

$x = 2$

$x^2 + 2x - 1 > x^2 - 4x + 4 - 3 \quad (-x^2)$

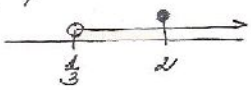
$2x - 1 > -4x + 1 \quad | + 4x + 1$

$6x > 2 \quad | : 6$

$x > \frac{2}{6}$

$x > \frac{1}{3}$

průnik



$\mathcal{K} = \{2\}$ (společný bod je jen jeden)

⑤ řeším v Z

$\frac{x-1}{4} + \frac{7-x}{2} > 3 \quad | \cdot 4 \quad \wedge \quad 2x - 3 - \frac{x^2+2}{3} \leq 2 - \frac{(3-x)^2}{3} \quad | \cdot 3$

$\mathcal{O} = \mathbb{Z}$!

$\mathcal{D} = \mathbb{Z}$.

$x - 1 + 2(7-x) > 12$

$x - 1 + 14 - 2x > 12$

$-x + 13 > 12 \quad | -14$

$-x > -1 \quad | : (-1)$

$x < 1$

$6x - 9 - (x^2 + 2) \leq 6 - (3-x)^2$

$6x - 9 - x^2 - 2 \leq 6 - (9 - 6x + x^2)$

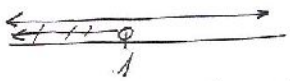
$-x^2 + 6x - 11 \leq 6 - 9 + 6x - x^2$

$0x - 11 \leq -3 \quad | + 11$

$0x \leq 8$

$10 \leq 8$),

platí $x \in R$



$\mathcal{K}' = (-\infty, 1) \quad [v R]$

$\mathcal{K} = \mathcal{K}' \cap Z = \{x \in Z ; x < 2\}$

$\mathcal{K} = \{0, -1, -2, \dots\} = Z_0^-$ (celá záporná včetně 0 – NEKLADNÁ CELÁ)